Conics: Hyperbolas

Saturday, March 10, 2012
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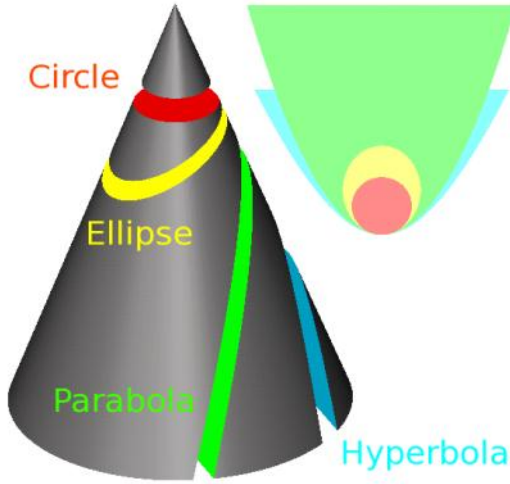


Here we come to the most complicated of the conics.
Standard form of a Hyperbola:

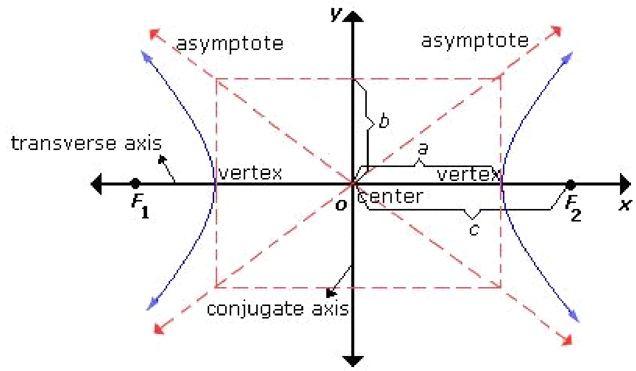
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \;\; or \;\; \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Horizontal Transverse　　　　Vertical Transverse

Note the differences: the placement of the x or y terms. This is because we now have a negative sign in between. Which ever term is above the a^2 term determines which way the Transverse axis runs.

Put the equation into the standard form for a Hyperbola.

$$4x^2 - 9y^2 - 32x + 36y - 8 = 0$$



Step 1: Group Terms.

$$4x^2 - 32x \quad - 9y^2 + 36y \quad = 8$$

Step 2: Complete the square.

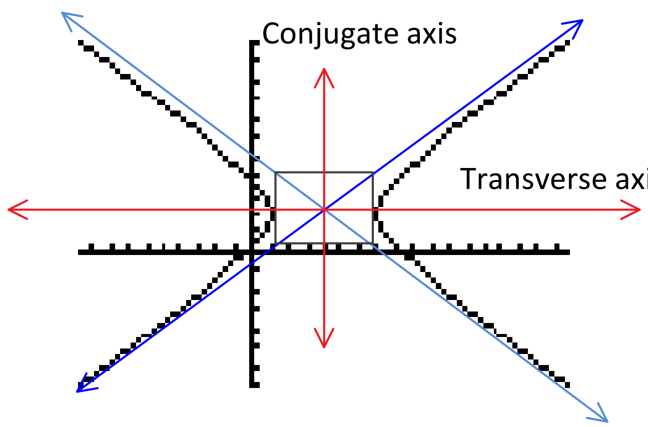
a) Remove the coefficents (factor out)

$$4(x^2 - 8x \quad) - 9(y^2 - 4y \quad) = 8$$
$$4(x^2 - 8x + 16) - 9(y^2 - 4y + 4) = 8 + 64 - 36$$
$$4(x-4)^2 - 9(y-2)^2 = 36$$

Step 3: Divide.

$$\frac{4(x-4)^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-4)^2}{9} - \frac{(y-2)^2}{4} = 1$$



Center: (4, 2)

$a^2 + b^2 = c^2 \quad 9 + 4 = 13 \quad c = \sqrt{13}$

Vertices: (7,2), (1,2)

Foci: $\left(4 \pm \sqrt{13}, 2\right) = (7.6, 2)\ and\ (.4, 2)$

Asymptotes: $y = 2 \pm \frac{2}{3}(x-4)$