Answers to

1) $f(x) = x^3 - 4x^2 + 3x + 2$

2) $f(x) = x^3 + 7x^2 + 17x + 15$

3) Factors to: $x(x-1)^2(x+3) = 0$
Roots: $\{0, 1 \text{ mult. } 2, -3\}$

4) Factors to: $x(x-5)(x-1) = 0$
Roots: $\{0, 5, 1\}$

5) Factors to: $x(x-5)(x-1)^2 = 0$
Roots: $\{0, 5, 1 \text{ mult. } 2\}$

6) Factors to: $x^2(x+5)(x+1) = 0$
Roots: $\{0 \text{ mult. } 2, -5, -1\}$

7) Factors to: $(x-5)(x^2-2) = 0$
Roots: $\{5, \sqrt{2}, -\sqrt{2}\}$

8) Factors to: $(x^2-3)(x^2+2) = 0$
Roots: $\{\sqrt{3}, -\sqrt{3}, i\sqrt{2}, -i\sqrt{2}\}$

9) Factors to: $(x+3)(x^2-3x+9) = 0$
Roots: $\left\{-3, \dfrac{3+3i\sqrt{3}}{2}, \dfrac{3-3i\sqrt{3}}{2}\right\}$

10) $(5x-3)(25x^2+15x+9) = 0$ See additional page
$x = 3/5$ $x = \dfrac{-3 \pm 3i\sqrt{3}}{125}$

11) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to -\infty$ as $x \to +\infty$

12) $f(x) \to +\infty$ as $x \to -\infty$
$f(x) \to +\infty$ as $x \to +\infty$

13) $f(x) \to -\infty$ as $x \to -\infty$
$f(x) \to +\infty$ as $x \to +\infty$

14) $f(x) \to +\infty$ as $x \to -\infty$ #15 Next page
$f(x) \to +\infty$ as $x \to +\infty$

15) ~~The dot next to the choice indicates that it is the answer~~.

16) Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
Factors to: $(x+2)(x^2-3) = 0$ deg=3 2 turns
(0,-6) = y-int

17) Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8$ deg = 3
1 turns
Factors to: $(x-2)(x^2+2x+4) = 0$ (0,-8)=y-int

deg=8 18) Possible rational roots:
7 turns $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100$
(0, 100) Factors to: $(x^2+2)(x^2-2)(x^2+5)(x^2-5) = 0$
y-int

19) Possible rational roots: deg=6 5 turns
$\pm 1, \pm 3, \pm 5, \pm 15, \pm 25, \pm 75$ (0,-75) y-int
Factors to: $(x^2+3)(x^2+5)(x^2-5) = 0$

20) $\{5\}$

21) $\{1\}$

22) $\{8\}$

23) $\{4, 1\}$

24) $g^{-1}(x) = -\dfrac{4}{x+1} - 2$

25) $f^{-1}(x) = -\dfrac{3}{x+2}$

26) Vertical Asym.: $x = -2$
Holes: None
slant Horz. Asym.: None changed
X-intercepts: $-3, 1$
Domain: All reals except -2

27) Vertical Asym.: $x = -2, x = 2$
Holes: $x = 0$
Horz. Asym.: $y = 0$
X-intercepts: None
Domain: All reals except $-2, 2, 0$

28) Vertical Asym.: $x = 4$
Holes: $x = -4$
Horz. Asym.: $y = -\dfrac{1}{3}$
X-intercepts: 2
Domain: All reals except $4, -4$

29) Vertical Asym.: $x = -2$
changed Holes: None
slant Horz. Asym.: None
X-intercepts: $-1, -3$
Domain: All reals except -2

30) Vertical Asym.: $x = 3$
Holes: $x = 2$
Horz. Asym.: None
X-intercepts: $0, 4$ slant
Domain: All reals except $3, 2$

31) Vertical Asym.: $x = -3, x = 1$
Holes: None
Horz. Asym.: $y = 0$
X-intercepts: None
Domain: All reals except $-3, 1$

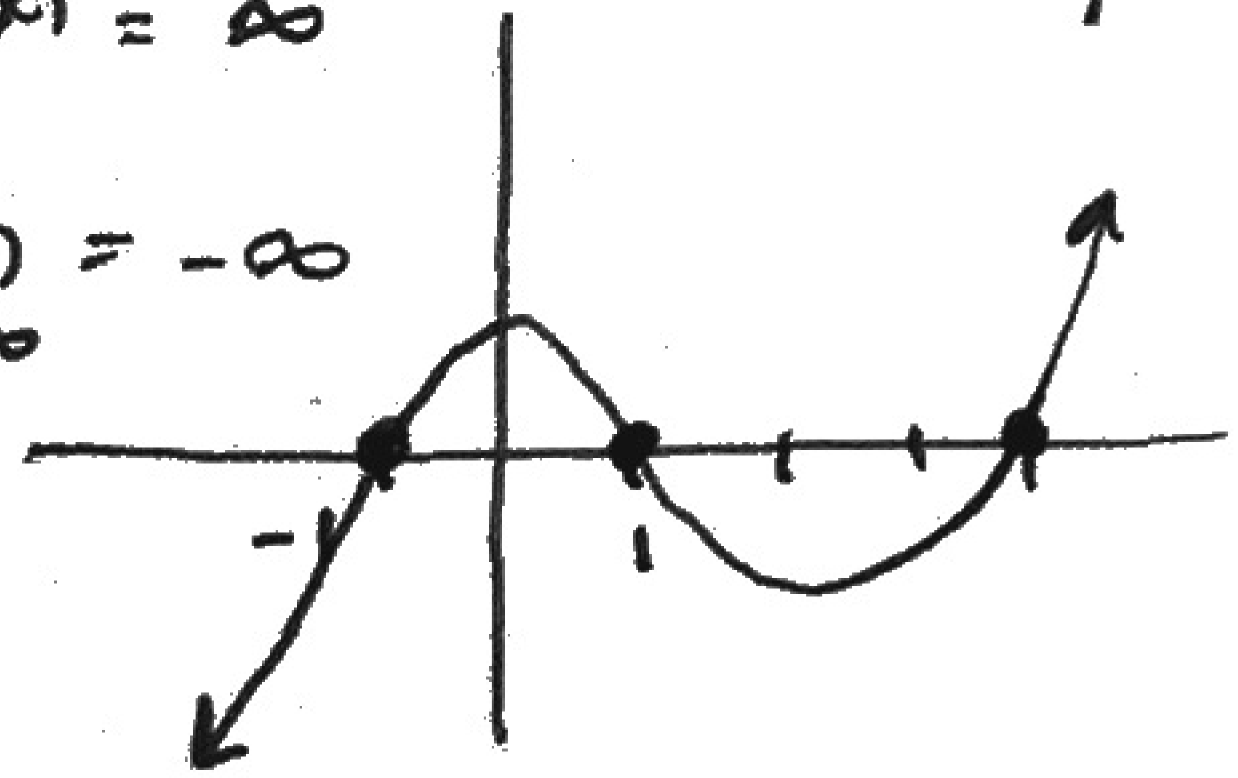
20, 22, 23, 26, 27, 28 are on separate pages.

15) Sketch the graph of the following functions:

A) $y=(x-1)^3(x-4)^5(x+1)^1$ $+/9$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$



w/ end behavior Bounce

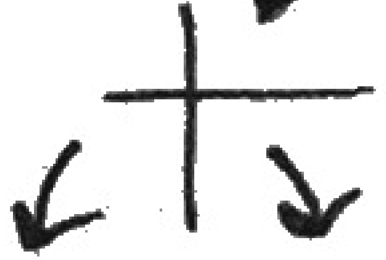
B) $y=-(x-2)^5(x-4)^3(x+1)^2(x-3)^2$

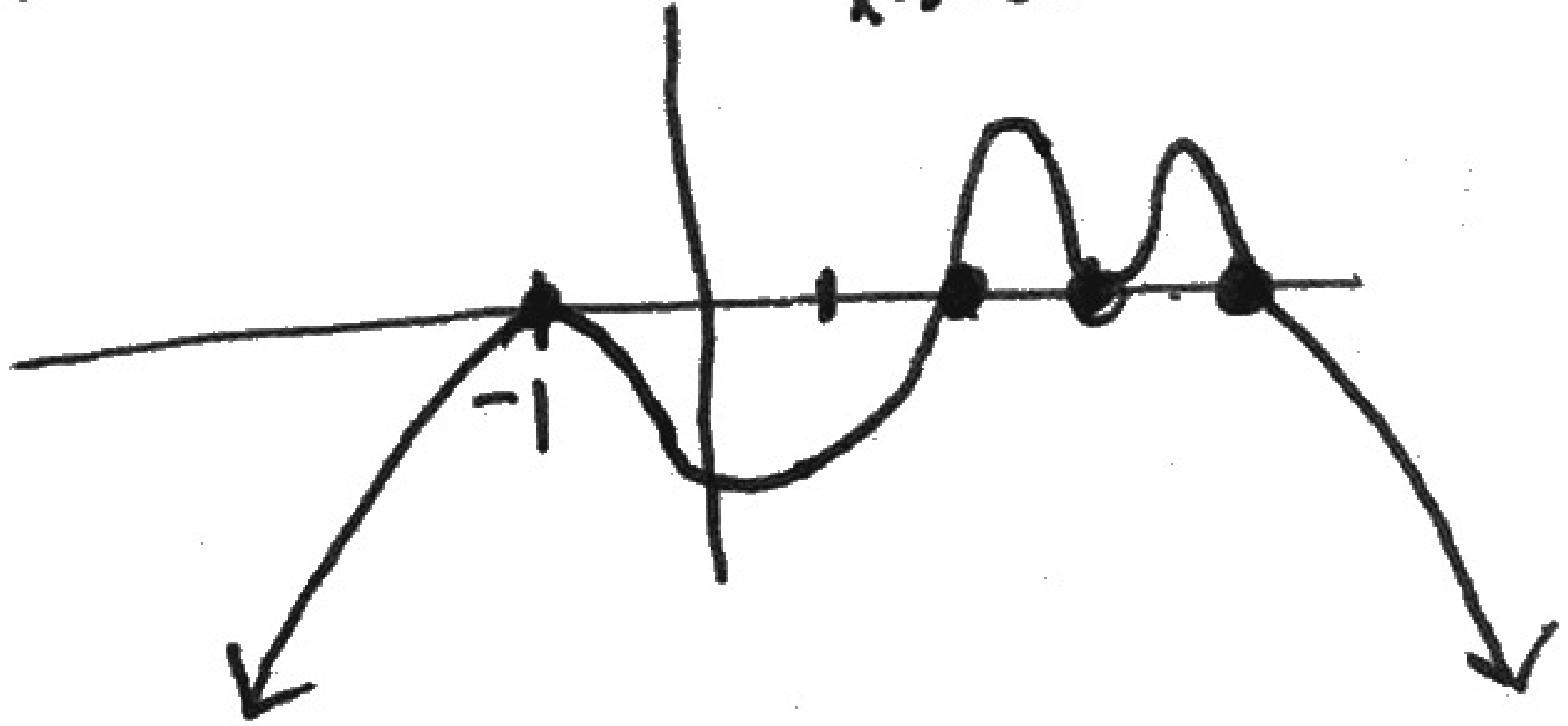
$5+3+2+2=12$

neg/even

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = -\infty$





10 $\frac{-15 \pm \sqrt{225-4(25)(9)}}{25^2} = \frac{-15 \pm \sqrt{-675}}{625}$ $\quad 25 \cdot 27$ $\quad \sqrt{-225 \cdot 3}$

$5x-3$ $\quad i15\sqrt{3}$

$\boxed{x = 3/5}$ $\quad \frac{\cancel{-15} \pm \cancel{15}i\sqrt{3}}{\cancel{5}(125)}$ $\quad \boxed{= \frac{-3 \pm 3i\sqrt{3}}{125}}$

20 $\frac{1}{6n} < \frac{1}{3n} - \frac{n-4}{6n}$ $\quad n \neq 0$ $\quad$ Dom: $(-\infty,0)\cup(0,\infty)$

$\frac{1}{6n} + \frac{n-4}{6n} < \frac{1}{3n}$

$\frac{1+n-4}{6n} < \frac{1\,(2)}{3n(2)} \Rightarrow \frac{n-3}{6n} < \frac{2}{6n}$ $\quad n-3 < 2$ $\quad \boxed{n < 5}$

22 $\frac{5}{v^2-4v} > \frac{1}{v} + \frac{1}{v^2-4v}$ $\quad v(v-4)=0$ $\quad v=0$ $\quad v=4$

$v \neq 0$ or 4 $\quad$ Dom: $(-\infty,0)\cup(0,4)\cup(4,\infty)$

$\frac{5}{v^2-4v} - \frac{1}{v^2-4v} > \frac{1\,(v-4)}{v\,(v-4)}$

$\frac{4}{v^2-4v} > \frac{v-4}{v^2-4v} \Rightarrow$ $\quad 4 > v-4$

$8 > v$ or $\boxed{v < 8}$

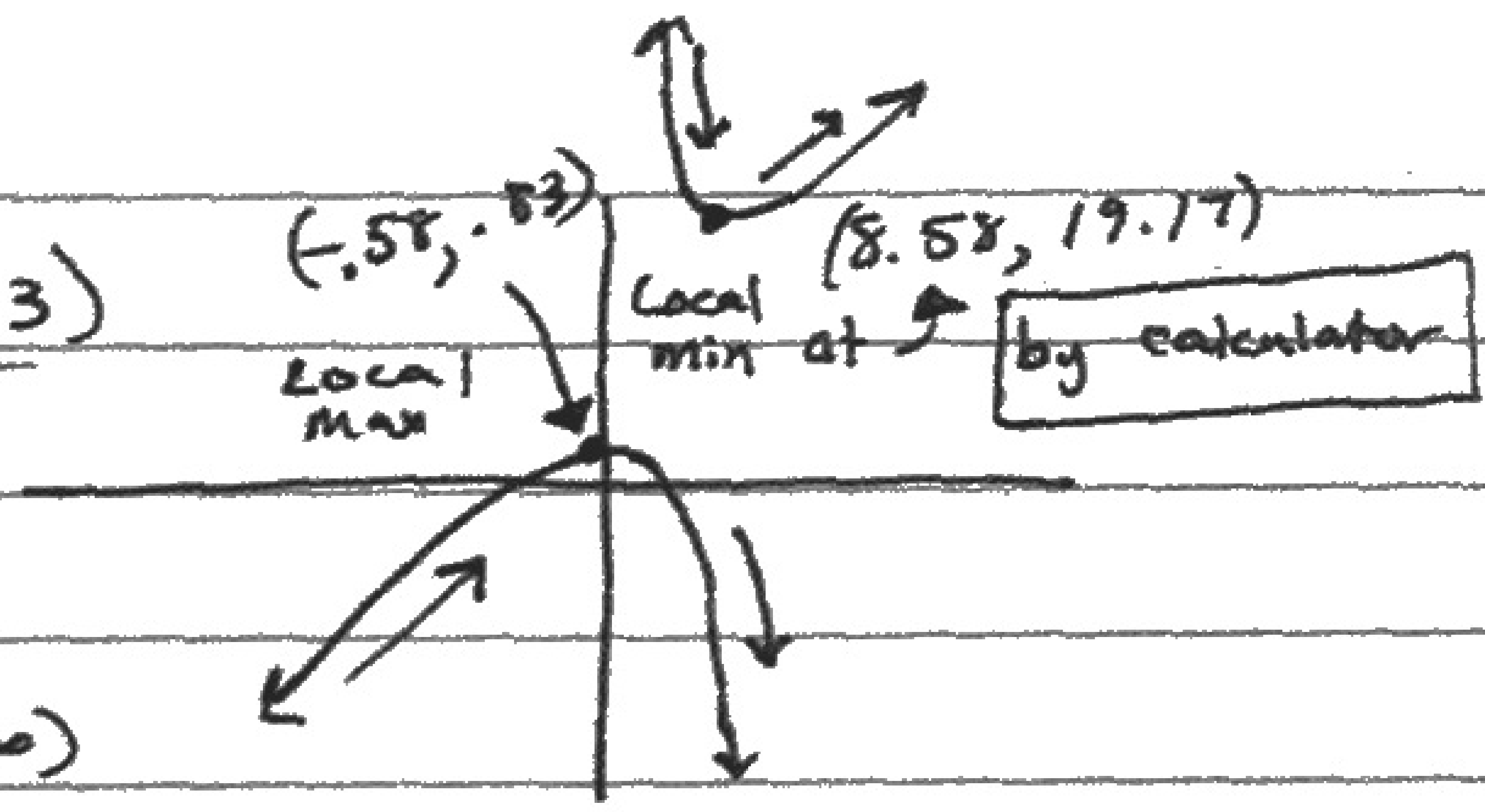
23 $\frac{(x)\,1}{(x)\,x-3} = \frac{1}{x^2-3x} + \frac{x-1\,(x-3)}{x\,(x-3)}$

$\frac{x}{x^2-3} = \frac{1}{x^2-3x} + \frac{x^2-4x+3}{x^2-3x} =$ $\quad x = 1 + x^2-4x+3$

$0 = x^2 - 5x + 4$ $\quad (x-4)(x-1)$

$\boxed{x = 4 \quad x = 1}$

26 $\dfrac{x^2+2x-3}{x-4} = \dfrac{(x-1)(x+3)}{(x-4)}$



X-vals Dom: $(-\infty,4)\cup(4,\infty)$

y-vals Range: $(-\infty,-.83)\cup(19.17,\infty)$

Discontinuous

X-vals Increasing $(-\infty,-.58)$ and $(8.58,\infty)$

x-vals Decreasing $(-.58,4)$ and $(4,8.58)$

Symmetry = neither

Local min + max on graph

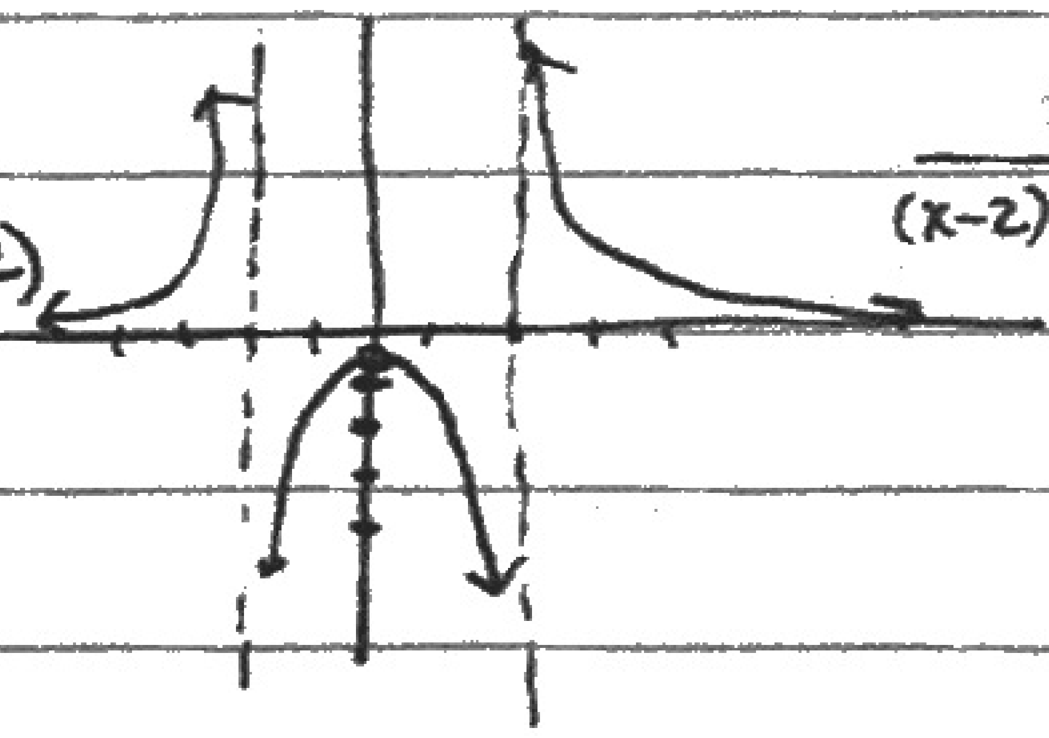
Asymptotes: $x=4$ (vertical)

$y=x+6$ (slant)

End behavior: $\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$

$$\begin{array}{r} x+6 \\ x-4\,\overline{)\,x^2+2x-3} \\ -x^2+4x \\ \hline 6x-3 \\ -6x+24 \\ \hline 0 \quad 21 \end{array}$$

27. $f(x) = \dfrac{2x}{x^3-4x} = \dfrac{2x}{x(x^2-4)} = \dfrac{2x}{x(x-2)(x+2)}$



$\dfrac{2}{(x-2)(x+2)} = \dfrac{2}{(-2)(2)}$

$\dfrac{2}{-4}$

$y=-1/2$

Dom: $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$

Range $(-\infty,-1/2)\cup(0,\infty)$

Discont.

Inc. $(-\infty,-2),(-2,0)$

Dec $(0,2),(2,\infty)$

Sym. even

Max/min = Local max $(0,-1/2)$

Asy/hole: vert $(x=-2)\ (x=2)$ hole at $x=0$

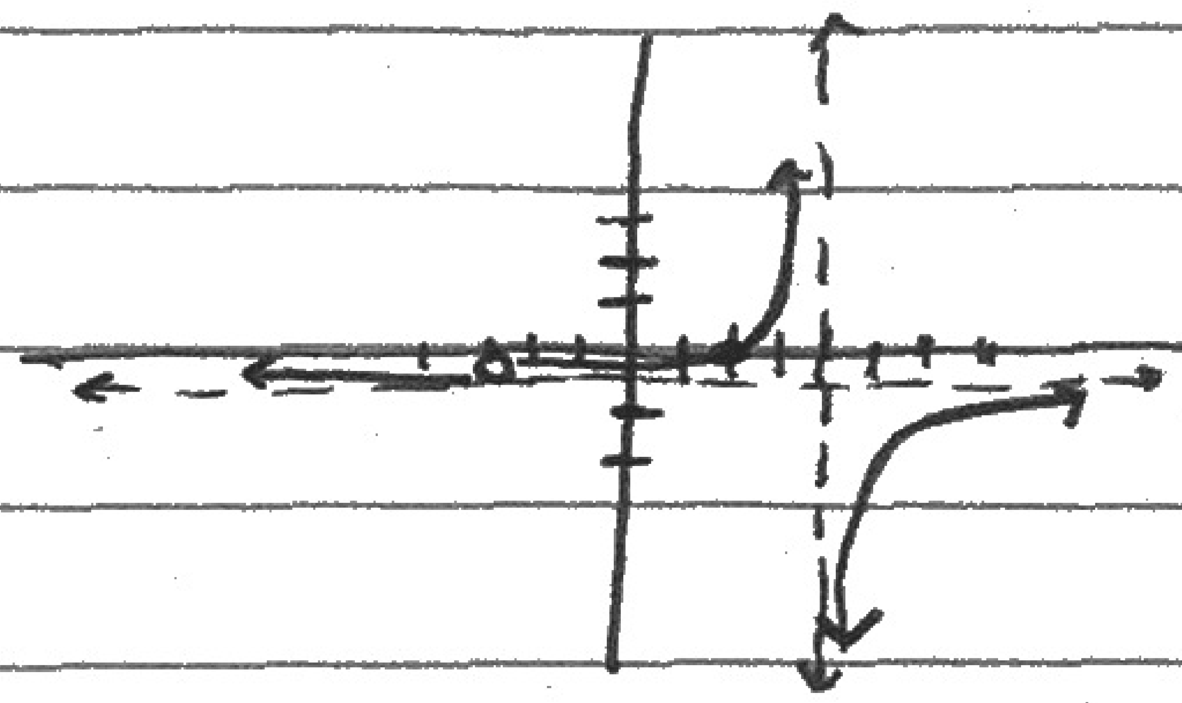
hor $y=0$

End behavior

$\lim_{x\to-\infty} f(x) = 0$ $\lim_{x\to\infty} f(x) = 0$

Fully Factored to see holes + Asymptotes

28 $f(x) = \dfrac{x^2+2x-8}{-3x^2+48} = \dfrac{(x-2)(x+4)}{-3(x^2-16)} = \dfrac{(x-2)(x+4)}{-3(x-4)(x+4)}$



Dom: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$

Range: $(-\infty,-\frac{1}{3})\cup(-\frac{1}{3},\infty)$

Discontinuous

Increasing: $(-\infty,-4)(-4,4)(4,\infty)$

Decreasing: None

Symmetry: Neither

Max/min: None

Asy/hole: Vert: $x=4$

Horiz: $y=-1/3$

Hole: @ $x=-4$ $\quad (-4,-\frac{1}{4}) \rightarrow \dfrac{(x-2)\cancel{(x+4)}}{-3(x-4)\cancel{(x+4)}} = \dfrac{(-4-2)}{-3(-4-4)}$

How to get (x,y) for holes...

Plug in ↳ $\dfrac{-6}{24} = -\dfrac{1}{4}$

End behavior

$\lim\limits_{x\to-\infty} f(x) = -\frac{1}{3} \qquad \lim\limits_{x\to\infty} f(x) = -\frac{1}{3}$

Notice how end behavior always relates to the horizontal asymptote.

Expect the problem on the test to be one with numbers that are not decimals like #26. More like #27.