

Sample Problems

Prove each of the following identities.

1. $\tan x \sin x + \cos x = \sec x$

2. $\frac{1}{\tan x} + \tan x = \frac{1}{\sin x \cos x}$

3. $\sin x - \sin x \cos^2 x = \sin^3 x$

4. $\frac{\cos \alpha}{1 + \sin \alpha} + \frac{1 + \sin \alpha}{\cos \alpha} = 2 \sec \alpha$

5. $\frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = 2 \tan x$

6. $\cos^2 x = \frac{\csc x \cos x}{\tan x + \cot x}$

7. $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

8. $\frac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

9. $\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

10. $1 - 2 \cos^2 x = \frac{\tan^2 x - 1}{\tan^2 x + 1}$

11. $\tan^2 \theta = \csc^2 \theta \tan^2 \theta - 1$

12. $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

13. $\frac{\csc \beta}{\sin \beta} - \frac{\cot \beta}{\tan \beta} = 1$

14. $\sin^4 x - \cos^4 x = 1 - 2 \cos^2 x$

15. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

16. $\frac{\sin^2 x + 4 \sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

17. $\frac{\cos x}{1 - \sin x} - \tan x = \sec x$

18. $\tan^2 x + 1 + \tan x \sec x = \frac{1 + \sin x}{\cos^2 x}$

Practice Problems

Prove each of the following identities.

$$1. \tan x + \frac{\cos x}{1 + \sin x} = \frac{1}{\cos x}$$

$$2. \tan^2 x + 1 = \sec^2 x$$

$$3. \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$$

$$4. \tan x + \cot x = \sec x \csc x$$

$$5. \frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

$$6. \tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$

$$7. \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x$$

$$8. \frac{\sec x - 1}{\sec x + 1} = \frac{1 - \cos x}{1 + \cos x}$$

$$9. 1 + \cot^2 x = \csc^2 x$$

$$10. \frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$$

$$11. \frac{\cot x - 1}{\cot x + 1} = \frac{1 - \tan x}{1 + \tan x}$$

$$12. (\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$$

$$13. \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$$

$$14. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$15. \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$$

$$16. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$17. \frac{\sin^2 x}{\cos^2 x + 3 \cos x + 2} = \frac{1 - \cos x}{2 + \cos x}$$

$$18. \frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$19. \frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$20. (\sin x - \tan x)(\cos x - \cot x) = (\sin x - 1)(\cos x - 1)$$