Vectors and Dot Product

Sunday, February 05, 2012
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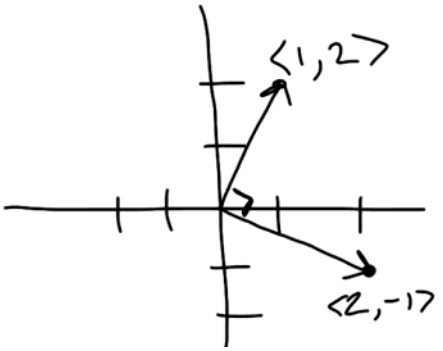
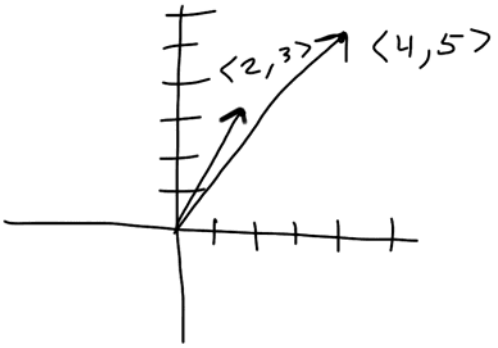


Dot Product:

$u \cdot v = u_1 v_1 + u_2 v_2$

Example: $\langle 4, 5\rangle \cdot \langle 2, 3\rangle \quad and \quad \langle 2, -1\rangle \cdot \langle 1, 2\rangle$

4(2) + 5(3) = 23 2(1)+(-1)(2) = 0



If you get zero as a result of a dot product then we know that the original vectors are perpendicular.

Finding Angles between any two vectors.

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

Example : $\langle 4, 3\rangle, \langle 3, 5\rangle$

$$\cos\theta = \frac{4(3)+3(5)}{5(\sqrt{34})}$$

$$\theta = \cos^{-1}\frac{27}{5\sqrt{34}} = 22.2°$$

Remember Magnitude:

$\|v\| = \sqrt{{v_1}^2 + {v_2}^2}$

$=\sqrt{4^2 + 3^2} = 5$

$=\sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$

Work

Work is force done multiplied by the distance over which the force was applied.

$W = F \cdot \overrightarrow{PQ}$
$W = \cos\theta \, \|F\| \left\|\overrightarrow{PQ}\right\|$

Example: A force of 45 pounds in the direction of $30°$ above the horizontal is required to slide a table across a floor. Find the work done

if the table is dragged 20 feet.

$W = \cos 30° \, (45)(20) = 779.42 \, \textit{foot-pounds}$